comprise $\sqrt{\Delta \overline{W}^2} = 7 \cdot 10^{-8}$ W•Hz$^{-1/2}$. For He I at $P = 2 \cdot 10^{-2}$ W•cm^{-2}, where helium boils on the sensor itself, fluctuations increase to $\sqrt{\Delta \overline{W}^2} = 8 \cdot 10^{-5}$ W•Hz$^{-1/2}$.

NOTATION

$R_{\square}$, resistance of square; R, resistance of temperature sensor; $\Delta R/\Delta T$, slope; I, bias current; P, power dissipation density; f, frequency; T, temperature; $\Delta \overline{T}^2$, mean square of temperature fluctuations; $\sqrt{\Delta \overline{T}^2}$, effective value of temperature fluctuations; $\sqrt{\Delta \overline{U}^2}$, effective value of noise voltage; $\Delta \overline{W}^2$, mean square power fluctuation; $\sqrt{\Delta \overline{W}^2}$, effective value of power fluctuation; G, heat transfer coefficient; R_T, thermal resistance.

LITERATURE CITED

1. N. A. Pankratov, G. A. Zaitsev, and I. A. Khrebtov, Radiotekh. Élektron., 15, No. 9, 1903 (1970).
2. I. M. Smith and M. W. P. Strandberg, J. Appl. Phys., 44, 2365 (1973).
3. V. L. Neuhouse and H. H. Edwards, Proc. IEEE, 52, 1191 (1964).
4. F. Sugawara, J. Appl. Phys. (Japan), 13, No. 2, 385 (1974).
5. G. Baker and D. E. Charlton, Infrared Phys., 8, 15 (1968).
6. G. J. Van Gurp, Phys. Rev., 166, No. 2, 436 (1968).
7. V. S. Golubkov and N. A. Pankratov, Inzh.-Fiz. Zh., 19, No. 1, 53 (1970).
8. N. A. Pankratov, G. A. Zaitsev, and I. A. Khrebtov, Prib. Tekh. Éksp., No. 3, 243 (1970); Cryogenics, 11, No. 2, 138 (1971).
9. J. I. Gittleman and S. Bozowski, Phys. Rev., 128, 646 (1962).

DYNAMIC DISLODGEMENT OF A VISCOPLASTIC LIQUID FILM

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UDC 532.135

A solution is obtained for the problem of shaking a viscoplastic film of limiting thickness off a flat or cylindrical surface moving in accordance with an exponential law.

A film of viscoplastic liquid on a vertical surface is characterized by the limiting static value of the thickness at which the film still will not flow under the influence of gravity. One method of dislodging such a film is to vibrate the wall. It is used, for example, in vibratory filters to remove the residue.

The limiting thickness of the film R_h is determined from the balance of friction and gravity forces. For a vertical cylindrical body

$$R_h = R\left(1 \pm \frac{2\tau_0}{\rho g R}\right)^{\frac{1}{2}}, \tag{1}$$

where R is the cylinder radius; τ_0 is the yield limit; ρ is density; g is the acceleration of gravity; the plus sign is taken for a film on the outside and the minus sign for a film on the inside surface of the cylinder.

Let us first consider the case $(\tau_0/\rho g R) \ll 1$. Then the cylindrical surface may be regarded as plane. We assume that the wall, coated with a film of limiting thickness, is moving in the direction of the gravity force at a constant velocity U_0 and at the initial instant of time ($t' = 0$) begins to decelerate, its velocity varying in accordance with the law

$$U = U_0 f(t'),$$

where $f(0) = 1$. As a result, two flow zones develop in the film: In the wall region, where the stresses exceed τ_0, we have viscoplastic flow; elsewhere the stresses are less than τ_0

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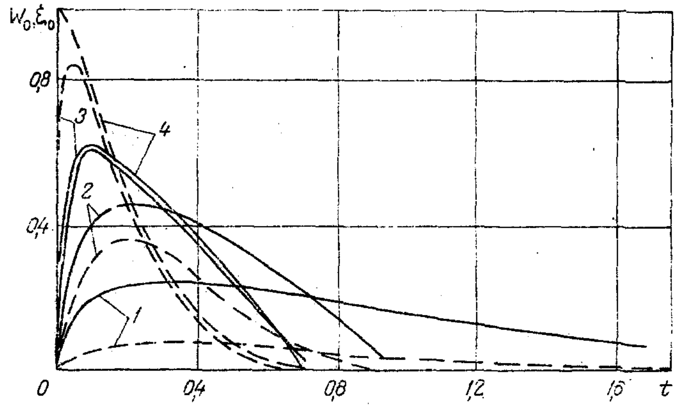

Fig. 1. Time dependence of the width of the viscoplastic flow zone (continuous curves) and of the velocity of the quasisolid core (dashed curves) for S = 2; A: 1) 1; 2) 5; 3) 50; 4) ∞.

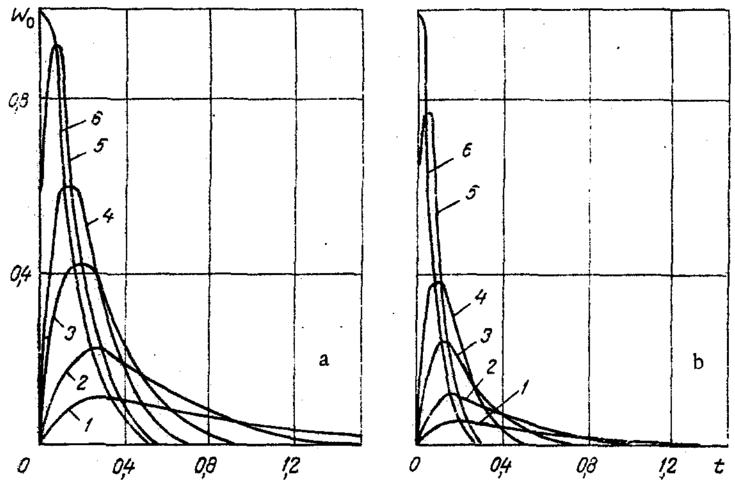

Fig. 2. Time dependence of the velocity of the quasisolid core for a film flowing off the inside (a) and outside (b) surfaces of a cylinder at S = 0.24; A: 1) 1; 2) 2; 3) 5; 4) 10; 5) 50; 6) ∞.

and the motion of the liquid is quasisolid. The case in which the wall stops instantaneously was examined in [1, 2].

For a Schwedoff–Bingham liquid

$$(\operatorname{sign}\tau)\,\tau = \tau_0 + \mu_p\,(\operatorname{sign}\dot{\gamma})\,\dot{\gamma}$$

($\dot{\gamma} = \partial u/\partial r$ is the shear rate, μ_p is the plastic viscosity) the problem reduces to the following equations of motion and boundary conditions

$$\frac{\partial u}{\partial t'} = \frac{\mu_p}{\rho}\cdot\frac{\partial^2 u}{\partial y^2} + g, \quad 0 \leqslant y < y_\sigma(t');$$
$$\frac{du_0}{dt'} = g - \frac{\tau_0}{\rho\,[h - y_\sigma(t')]}, \quad y_\sigma(t') \leqslant y \leqslant h;$$
$$u\,[y_\sigma(t'), t'] = u_0(t'), \quad \frac{\partial u}{\partial y}[y_\sigma(t'), t'] = 0; \tag{2}$$
$$u(0,t') = U = U_0 f(t'), \quad u(y,0) = U_0;$$
$$u_0(0) = U_0, \quad y_\sigma(0) = 0.$$

Here $h = \tau_0/\rho g$ is the film thickness; $y_\sigma(t')$ is the coordinate of the moving boundary of the viscoplastic and quasisolid zones; $u_0(t')$ is the velocity of the quasisolid core of the flow.

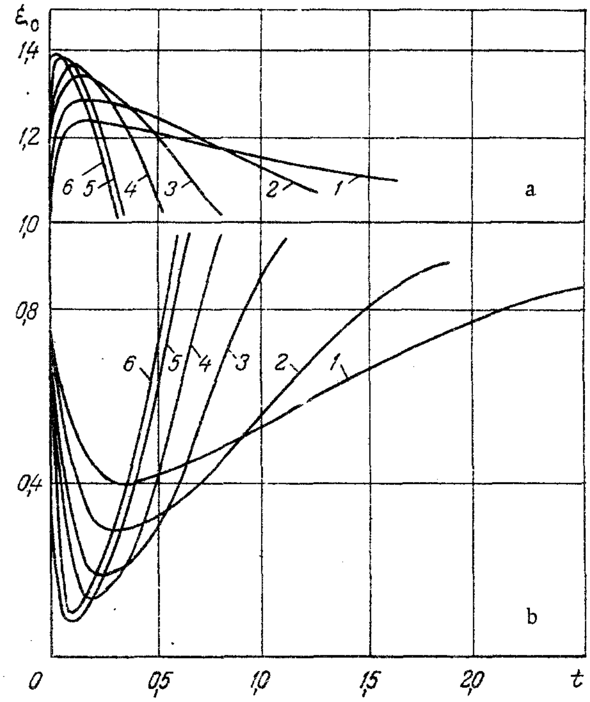

Fig. 3. Time dependence of the coordinate of the boundary of the quasisolid zone for S = 0.25; A: 1) 1; 2) 2; 3) 5; 4) 10; 5) 50; 6) ∞; a) outside surface; b) inside surface.

We introduce the dimensionless variables and parameters

$$\xi = \frac{y}{h},\ \xi_0 = \frac{y_\sigma}{n},\ v = \frac{u}{U_0},\ v_0 = \frac{u_0}{U_0},\ t = \frac{\mu_p t'}{\rho h^2},\ S = \frac{\tau_0 h}{\mu_p U_0}.$$

Then system (2) takes the form

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial \xi^2} + S \quad 0 \leqslant \xi < \xi_0; \tag{3}$$

$$\frac{dv_0}{dt} = \frac{S\xi_0}{1-\xi_0},\ \xi_0 \leqslant \xi \leqslant 1; \tag{4}$$

$$v(\xi_0, t) = v_0(t),\ \frac{\partial v}{\partial \xi}(\xi_0, t) = 0,\ v(0,t) = f(t); \tag{5}$$

$$v(\xi, 0) = 1,\ v_0(0) = 1,\ \xi_0(0) = 0. \tag{6}$$

Finding the exact solution presents serious mathematical difficulties; accordingly we will seek an approximate solution. We approximate the velocity profile by means of the expression

$$v(\xi,t) = \begin{cases} f - 2(f - v_0)\dfrac{\xi}{\xi_0} + (f - v_0)\dfrac{\xi^2}{\xi_0^2}, & 0 \leqslant \xi < \xi_0, \\ v_0, & \xi_0 \leqslant \xi \leqslant 1, \end{cases} \tag{7}$$

which satisfies boundary conditions (5).

We satisfy Eq. (3) in the mean:

$$\int_0^{\xi_0} \frac{\partial v}{\partial t}\,\partial\xi = \int_0^{\xi_0} \left(\frac{\partial^2 v}{\partial \xi^2} + S\right) d\xi.$$

After integration, using (4) and (7), we arrive at the following equation for the boundary of the quasisolid and viscoplastic zones:

$$\frac{d\xi_0}{dt} = \frac{6}{\xi_0} + \frac{3S\xi_0}{f - v_0} + \frac{\xi_0}{f - v_0}\cdot\frac{df}{dt} + \frac{2S\xi_0}{(f - v_0)(1 - \xi_0)}. \tag{8}$$

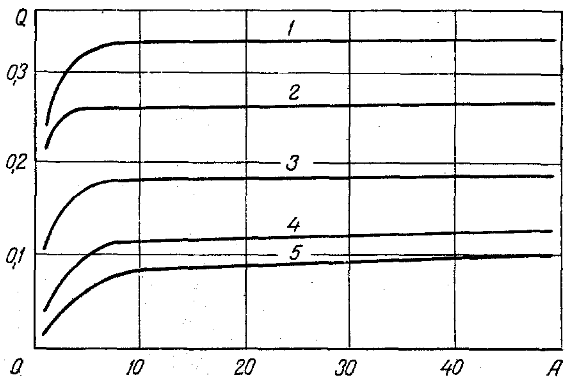

Fig. 4. Dependence of the mass of liquid shaken off on A for: 1) S = 0.25, KR = 0.5 (inside surface); 3) S = 0.25, KR = 0.5 (outside surface); 2) S = 0.25; 4) 5; 5) 10 (plate).

System of equations (4), (8) for $v_0(t)$ and $\xi_0(t)$, with initial conditions (6), was solved numerically on a Minsk-22 computer for the case

$$f = \exp(-At). \tag{9}$$

Some of the results are presented in Fig. 1. The viscoplastic region initially increases reaches a maximum, and then decreases. The velocity of the quasisolid zone relative to the plate $W_0 = v_0 - \exp(-At)$ increases from zero to a maximum, then falls monotonically. As the deceleration parameter A increases, the velocity maximum grows and is shifted to the left, tending in the limit to the case in which the wall stops instantaneously.

The mass of liquid shaken off the wall per unit width of film is given by

$$Q = \int_0^\infty \int_0^1 (v - f)\, d\xi dt = \int_0^\infty (f - v_0) \left(\frac{\xi_0}{3} - 1 \right) dt \tag{10}$$

and is represented in Fig. 4. As the parameter A increases, so does the amount of liquid shaken off. The rate of increase is particularly significant up to A ≈ 10, after which the increase in Q is very slight. Thus, for example, the values of Q for A = 10 at S = 0.25 and 10 are 96.5 and 79%, respectively, for instantaneous stopping of the wall. Increasing the plasticity parameter S reduces the amount of liquid shaken off.

When $(\tau_0/\rho g R) \sim 1$ the curvature of the surface must be taken into account. In order to solve the problem of a film of viscoplastic liquid shaken off a vertical cylindrical surface we use the method described above.

We then obtain

$$v(\xi, t) = \begin{cases} \dfrac{f\xi_0^2 + v_0 - 2v_0\xi_0}{(\xi_0 - 1)^2} - 2\xi_0 \dfrac{f - v_0}{(\xi_0 - 1)^2}\, \xi + \dfrac{f - v_0}{(\xi_0 - 1)^2}\, \xi^2; \\ v_0 \end{cases} \tag{11}$$

moreover,

$$\frac{d\xi_0}{dt} = \pm \frac{3S\ln\xi_0}{v_0 - f} - \frac{6\xi_0 \ln\xi_0}{(1 - \xi_0)^2} - \frac{12}{1 - \xi_0} - \frac{1 - \xi_0}{v_0 - f} \cdot \frac{df}{dt} + \frac{1 - \xi_0}{v_0 - f} \left(\frac{S}{KP} + \frac{4S\xi_0}{\pm\xi_0^2 + 2KR \mp 1} \right), \tag{12}$$

$$\frac{dv_0}{dt} = \frac{S}{KR} - \frac{2S\xi_0}{\pm\xi_0^2 \mp 1 + 2KR}, \tag{13}$$

$$\xi_0(0) = 1, \; v_0(0) = 1. \tag{14}$$

The mass of liquid shaken off is given by the expression

$$Q = \frac{\pi}{6} \int_0^\infty (v_0 - f)\,[12KR \mp 4 \pm (\xi_0 + 1)^2]\, dt. \tag{15}$$

Here $\xi = r/R$, $\xi_0 = r_\sigma/R$, $KR = \tau_0/\rho gR$ is the surface-curvature parameter. The upper sign in expressions (12), (13), and (15) relates to the inside-face problem, the lower sign to the outside-face problem.

The results of a numerical integration are presented in Figs. 2 and 3. The behavior of the velocity W_0 of the quasisolid core relative to the wall and the zone boundary ξ_0 is qualitatively similar to the plane case. The other parameters being equal, the maximum values W_0^{max} and $|1 - \xi_0^{max}|$ for a film on the inside surface of a cylinder are higher and displaced to the right as compared with the case of flow off the outside surface. In this connection the mass of liquid shaken off is greater for the film flowing off the inside surface of the cylinder (Fig. 4). As the deceleration parameter A increases, so does the mass of liquid shaken off. The rate of increase is particularly significant up to $A \approx 10$.

LITERATURE CITED

1. V. I. Baikov, Izv. Akad. Navuk BSSR, Ser. Fiz.-Énerg. Navuk, No. 3 (1974).
2. Z. P. Shul'man and V. I. Baikov, Izv. Akad. Navuk BSSR, Ser. Fiz.-Énerg. Navuk, No. 1 (1976).

VARIATIONAL SOLUTION OF EQUATION OF NONLINEAR MASS AND ENERGY TRANSFER

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The use of a variational principle of Hamilton type is considered for problems of nonlinear mass transfer in a semibounded plate with constant and variable diffusional properties.

In complex cases associated with heat and mass transfer in systems with chemical transformations or polymerization processes, in biological systems, and in special cases of catalytic processes, significant deviation from the Fourier and Fick laws is observed. Processes of heat and mass transfer of this kind may be mathematically described by an equation of the form

$$\frac{\partial \varphi}{\partial \tau} + \operatorname{div}(\vec{v}\varphi) + \tau^* \frac{\partial^2 \varphi}{\partial \tau^2} = \operatorname{div}[k(\varphi)(\operatorname{grad}\varphi)^n] + F(\varphi), \quad n \leqslant 1. \tag{1}$$

This equation may be obtained on the assumption that the flux of material is determined by an expression of the form [1]

$$\vec{j} = -k(\varphi)(\operatorname{grad}\varphi)^n - \tau^* \dot{\vec{j}}.$$

In the case of heat transfer, φ represents the thermal energy; in the case of mass transfer, the concentration. For the heat-transfer equation $F(\varphi)$ is a heat source or sink and for the mass-transfer equation a mass source or sink due to chemical transformations.

Consider the case of mass and energy transfer in a semibounded plate with variable diffusional properties in the presence of a chemical reaction; in this case, Eq. (1) takes the form

$$\frac{\partial C}{\partial \tau} = \frac{\partial}{\partial x}\left[D^*(C)\left(\frac{\partial C}{\partial x}\right)^n\right] + kC^m, \quad n \leqslant 1, \quad m = 1, 2, 3, \tag{2}$$

where k is the rate constant of the chemical reaction; m is the order of the chemical reaction. In the general case it is expedient to assume that the order of the reaction may be either integral or fractional.

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